Quantum computation by optically coupled steady atoms/quantum-dots inside a quantum electro-dynamic cavity

Prabhakar Pradhan[1*], M.P. Anantram[2] and Kang L. Wang[1]

[1]*Electrical Engineering Department, University of California, Los Angeles, CA 90095*
[2]*Nasa Ames Research Center, Moffett Field, CA 94035*

We present a model for quantum computation using n steady 3-level atoms or 3-level quantum dots, kept inside a quantum electro-dynamics (QED) cavity. Our model allows one-qubit operations and the two-qubit controlled-NOT gate as required for universal quantum computation. The n quantum bits are described by two energy levels of each atom/dot. An external laser and n separate pairs of electrodes are used to address a single atom/dot independent of the others, via Stark effect. The third level of each system and an additional common-mode qubit (a cavity photon) are used for realizing the controlled-NOT operation between any pair of qubits. Laser frequency, cavity frequency, and energy levels are far off-resonance, and they are brought to resonance by modifying the energy-levels of a 3-level system using the Stark effect, only at the time of operation.

03.67.Lx, 73.20.Dx, 42.50.Md

A computer, which follows quantum mechanical principles, has significant advantages over a classical computer [1,2]. Implementing a quantum computer is based upon the implementation of basic quantum units called quantum bits (two-level systems) and communication among them. Logical operations of a quantum computer can be decomposed into a series of an arbitrary one-qubit rotation plus a two-qubit controlled-NOT operation, thus this set of operations makes a universal quantum computer [3,4]. A similar set in which the controlled-NOT is replaced by the controlled-phase-shift gate [$|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow |01\rangle$, $|10\rangle \rightarrow |10\rangle$, and $|11\rangle \rightarrow -|11\rangle$] is also universal [5]. n two-level systems can have 2^n highly entangled (phase coherence) states and a quantum computer takes advantages of performing unitary transformations in a parallel manner on these 2^n classical strings.

The main difficulties in implementing a quantum computer are the contradicting demands in terms of interaction with the environment. On one hand, a strong *controlled* interaction is desired in order to operate the computing algorithm (to switch the state of the qubits), but on the other hand *uncontrolled* interactions are strongly undesired since they cause decoherence of the qubits and hence loss of computing ability. All quantum systems lose their coherence after some time due to non-zero coupling with the environment. Thus the above problem is usually expressed as the need to increase the ratio between the decoherence time and the switching time: a quantum computer must perform *all* calculations within the decoherence time of the qubit.

Several theoretical and experimental attempts are currently ongoing to realize simple gates (such as the controlled-NOT gate between two qubits). The most realistic ones at the moment are ion-trap [5], liquid NMR [6], and cavity-QED [3,7,8]. However, serious problems in scaling these systems, and/or in addressing particular qubits create the need for better suggestions or major modifications of these implementations.

The first interesting experiments were done on cavity-QED systems [8]. The cavity-QED computation model is based on the idea of having two types of qubits (atoms and cavity-modes) and it was found very useful in implementing various gates. However, the requirement of mechanical control of atoms makes this model less desirable for quantum computing: the interaction time is controlled by the physical motion of the atoms inside the cavities, and having enough control to let the atom enter a cavity several times (whenever required by the algorithm) is difficult.

The long decoherence time of liquid-NMR and ion trap systems, and easy control of the qubits make these systems a serious candidate for quantum computation. But the problem is in scaling these systems with the increasing numbers of qubits. Like, in case of bulk liquid-NMR, the signal from the system decreases exponentially with the numbers of qubits. Thus the number of qubits comprising the quantum computer has to be small. The ion-trap computation model is based on interaction via a common-mode qubit and it has two main problems: (a) the addressing of an individual qubit by a separate laser directed to each ion is an idea which cannot be implemented yet. (b) the use of only one type of two-qubit interaction—interaction via one common mode. This problem prevents the possibility of running simultaneous several gate operations.

Proposals for a solid state [9,10] and solid-state NMR [11] devices based on nanotechnology, might be more promising for the far future. But even single qubit systems have not been implemented in these systems due to the difficulties in creating and controlling such a single qubit.

Clearly, more candidates for realization of quantum computing devices are still needed, with the hope of a more diverted experimental effort. Such an effort, mainly in the direction of solid-state devices, but combining ideas from existing implementations, might lead to a sys-

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tem where single qubits can be addressed, scaling to large number of qubits made possible, and in the future, may be even a system where fault-tolerant computation can be performed.

This paper suggests a model of a quantum computer that combines the advantages of other models [12]. We shall show how to implement the universal set of gates containing the controlled-phase-shift and the arbitrary one qubit rotation.

As a first step we suggest a hypothetical model of atoms fixed in a cavity, and a pair of electrodes "directed" around each atom to control its energy level-spacing. In this first step, we combine the use of a common mode as in the ion-trap computation model [5], with the two types of qubits as suggested by cavity-QED models [3,7,8], to obtain better control of addressing a single qubit. Unfortunately, fixing atoms for the required time scales is not yet realistic. In general, atoms are in motion in all cavity QED experiments.

In the second step, we suggest replacing the atoms by quantum dots, so the idea of "fixing" the qubits becomes more realistic. The technical ability of putting a single qubit in a single quantum dot, and the technical ability of putting a quantum dot in a cavity exist separately. Combining them together (while demanding also that the cavity is highly reflecting) is far from the ability of current experiments, but we hope to motivate this direction by showing that the computation model we obtain is very promising. Recently, it has been experimentally shown [13] that a single electron can be controlled in a quantum dot; The dot size and dielectric modulation are however large $(.5\mu m)^3$.

A sketch of the model for the proposed quantum-computer (with steady atoms) is shown in Fig.1. Atoms are kept steady along the axis of the cavity. An external laser source is accessible to all atoms, and is directed perpendicular to the cavity axis. Electrodes around each atom (which we refer as "Stark plates") are used to control its level spacing via the Stark effect, and are perpendicular to the laser and the cavity axis. When required in the protocol, a strong electric field is applied to the atom by changing the voltage on the electrodes. This field changes the energy level separation (a thorough study has been done for Rydberg atoms in Ref. [14,15]). The required electric field can be calculated easily once the energy levels and the wave functions of the system are chosen. We will assume that the on/off switching of the electric field is but slow such that the change in the original wave function is insignificant. At the same time, it must be fast relative to the time steps of the computation. The applied DC field has to be a fraction of the order of the atomic energy level separations.

Quantum bits of the computer are described by the ground state ($|g\rangle$), and the first excited state ($|e_0\rangle$) of the atom; a third level ($|e_1\rangle$) is used for a controlled phase shift operation. Rotation of an individual qubit is

achieved by applying a laser pulse to all atoms, while only one the qubit undergoing transformation is on-resonance with the laser frequency, and others are far off-resonance.
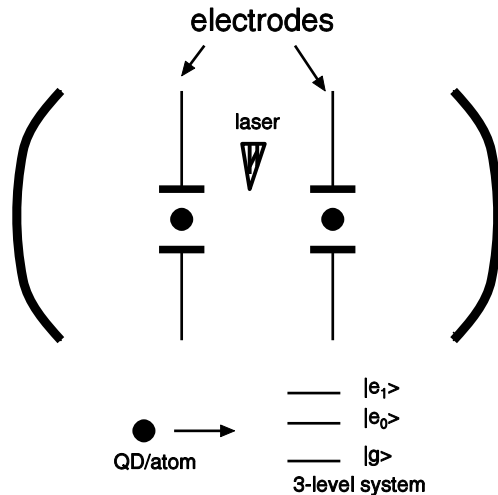

FIG. 1. Atoms/QDs are kept along the axis of a perfectly reflecting cavity. Electric plates are kept attached around each of the atoms/QDs to control the energy levels via Stark effect. A laser source pointing toward all atoms is kept perpendicular to both, i.e., the cavity and the Stark plate's axes.

Communication between any two qubits is done by a common mode cavity photon as described now. The photonic mode is in its ground state (zero photons) and a maximum of one cavity photon is present at the time of interaction between two qubits. The cavity's 0-photon and 1-photon states are defined by $|0\rangle$ and $|1\rangle$ respectively. The atomic levels are kept far off-resonance with respect to the resonant frequency of the cavity, to avoid undesired interaction (in which a transition from excited state to ground state takes place while emitting a photon to the cavity). Desired energy levels are brought into resonance with the cavity by changing the Stark field only at the time of logic operations. To perform a controlled operation between any two qubits, we do the following: (a) The state of the first qubit and the vacuum state of the cavity are swapped. (b) the new cavity state is used to perform a controlled operation with another qubit; the third level of the atom is used for that purpose, yielding a controlled-phase-shift gate. (c) Finally, the cavity state is again swapped with the first qubit, so the cavity is back into its vacuum state, and the controlled operation between the two qubits is completed.

We assume that the time for a significant far off-resonance evolution is huge compare to the on-resonance evolution time. We also assume that the cavity is of high-quality and has almost perfect reflecting walls, so that the decoherence time for the cavity mode is much

larger than the time between the two required swap operations. The frequency of the laser pulses is off-resonant with the cavity.

We will describe in detail the Hamiltonian leading to the single qubit rotations and the controlled-phase-shift operations. This is done by taking into account the fact that $|g\rangle \to |e_0\rangle$ and $|e_0\rangle \to |e_1\rangle$ are allowed dipole transitions, but $|g\rangle \to |e_1\rangle$ is not an allowed transition due to the definite parity of the wave function.

Let $\omega_{g;e_0}$ be the level separations of the qubit (a similar definition applies for $\omega_{e_0;e_1}$ and $\omega_{g;e_1}$). For the atomic levels with definite parity, we assume that the levels are chosen so that the difference frequencies $\omega_{g;e_0}$ and $\omega_{e_0;e_1}$ are nearly the same. We treat this case here, but one can easily treat the case where other transitions are allowed or forbidden.

In the following, we describe the steps to obtain the necessary operations involving only one atom at a time, by bringing its levels to be on-resonance with the laser frequency or the cavity mode. The other atoms are kept far off-resonance to avoid their interactions. If the initial levels are such that $g;e_0$, and $e_0;e_1$ are the allowed transitions, and $\omega_{e_0;g} < \omega_{e_0;e_1}$, then one way to choose the cavity and the laser frequencies are such that $\omega_{e_0;g} < \omega_{e_0;e_1} < \omega_l < \omega_c$.

By increasing the level separations, the qubit can be brought to be on-resonance with the laser. This increase in level separation must be significant enough that the interaction of off-resonant atoms with the laser is insignificant. By increasing the level separations further, the qubit is brought to be on-resonance with the cavity photon. Each level separation increases with the applied electric field. While increasing the level separation $\omega_{e_0;g}$, the level separation $\omega_{e_0;e_1}$ will first come to resonance with the cavity. But we will assume that the switching time is much smaller than the inverse Rabi frequency of the atom-cavity system such that there is practically no effect of this resonance crossing.

One qubit rotation is performed by changing the atomic levels so that $\omega_l = \omega_{e_0} - \omega_g$ and applying the laser pulse. The laser and qubit involved interact on resonance (but ω_l is off resonance with the cavity and with other qubits). The Hamiltonian for the atomic levels in the presence of the laser field is [5]:

$$H_1 = \frac{\Omega_l}{2}\left[\sigma_+ e^{-i\phi} + \sigma_- e^{i\phi}\right].$$

Where $\sigma_+ = |e_0><g|$, $\sigma_- = |g><e_0|$, ϕ is the phase factor of the laser at the location of the basic unit, and Ω_l is the Rabi frequency due to the laser $\Omega_l = E_0 \mu_{ge_0}$, where μ_{ge_0} is the dipole moment for $|g> \to |e_0>$ transition and E_0 is the strength of the electric field.

If the interaction time between the laser pulse and the qubit is $t = \frac{k\pi}{\Omega_l}$, then the time evolution operator is

$$\hat{V}_m^k(\phi) = exp[-ik\frac{\pi}{2}(\sigma_+^m e^{-i\phi} + \sigma_-^m e^{i\phi})]$$

The process is an energy non-conserving process, and the system is fed energy from the laser field.

The Jaynes-Cumming Hamiltonian [7] for a 2-level system, which is on-resonance with the cavity photon is described by:

$$H_2 = i\frac{\Omega_c}{2}\left[\sigma_+ \hat{a}^\dagger - \sigma_- \hat{a}\right]$$

Where a and $a^\dagger$ are the annihilation and creation operators for common mode photon, and Ω_c is the photon Rabi frequency of the cavity-atom system.

If the interaction time between the laser pulse and the qubit is $t = \frac{k\pi}{\Omega_c}$, then the time evolution operator is

$$\hat{U}_m^k(\phi) = exp[-ik\frac{\pi}{2}[i\sigma_+^m \hat{a}^\dagger - i\sigma_-^m \hat{a}]].$$

To get a control-phase-shift between two qubits (two atoms/QDs, say m and n such that m is the control and n is the target), we need two types of cavity-atom operations: A π pulse for obtaining the swap operation, where the qubit levels $g;e_0$, and the cavity levels are used and a 2π pulse using the third level and the cavity photon to obtain the atom-cavity controlled-phase-shift.

The operation is done in three steps:

(1) The levels $|g\rangle_m$ and $|e_0\rangle_m$ of the mth atom are brought into resonance with the cavity. The system is let to evolve on-resonance with the cavity for a time equal to π/Ω_c. At the end of this, a SWAP operation the state of the atom with the state of the cavity (which is the vacuum state) occurs. After the interaction, the mth atom is in its ground state.

(2) The states $|g\rangle_n$ and $|e_1\rangle_n$ are brought to resonance with the cavity and let to evolve for a time equal to $2\pi/\Omega_c$. The result is that the state doesn't change if there is no photon in the cavity: $|g_n 0\rangle \to |g0_n\rangle$, $|e_{0n}0\rangle \to |e_{0n}0\rangle$; Also, there is no change if the nth atom is in its ground state: $|g1\rangle \to |g1\rangle$; however, if there is a photon in the cavity, and the nth atom is in the excited state, it gets a phase $|e_{0n}1\rangle \to -|e_{0n}1\rangle$ (since it is a spinor).

(3) The mth atom and the cavity are brought into resonance and the system is let to evolve for a time equal to π/Ω_c. At the end of this, a SWAP operation the state of the mth atom with the state of the cavity (which is the vacuum state) occurs.

A π pulse ($k = 1$) is given between the levels $|g\rangle_m$ and $|e_0\rangle_m$ by bringing these two levels (of the m'th atom) on resonance with the cavity to SWAP again their states. After the interaction, the cavity is back in the vacuum state, but the state of the qubits change.

A crucial issue in this model is the relative time scale between the cavity on-resonance and off-resonance with the 2-level system. When the cavity is off-resonance in presence of a photon, a dressed state evolves. The relative time scale of the evolution is [7]:

$$\frac{\tau_{off-resonance}}{\tau_{on-resonance}} = \sqrt{(\frac{\omega_c - \omega_{ge_0}}{\Omega_c})^2 + 1}$$

where ω_c is the cavity frequency $\omega_{ge_0} = \omega_{e_0} - \omega_g$ and Ω_c is the Rabi frequency of the atom due to the cavity photon. The vacuum off-resonance phase evolution, $\Omega_c^2/(\omega_c - \omega_{e_0 g})$ must be small enough to make the off-resonant evolution insignificant [7]. Another way to get rid of the vacuum off resonance evolution is by nullifying the extra phase evolution by additional logic operations or by taking into account the phase in every step of operations.

The spontaneous emission time is quite low or negligible for a trapped atom. The ratio of decoherence time to the time required for a single operation is $\approx 10^6$. That is, 10^6 pulses can be applied within the coherence time. In case of a Rydberg atom, for $50 \to 51$ transition, $\omega_{ge_0} = 5 * 10^{10}$ Hz, the cavity length is ≈ 1 cm, $\Omega_c \approx 4 * 10^5$ Hz, and $\delta = 4 * 10^6$ Hz, where δ is the detuning, i.e., $\omega_{e;g} - \omega_l$.

In the case of a quantum dot with transition energy $\omega_{ge_0} \approx 1meV$ (1 THz), $\Omega_c \approx 10^8$ Hz and the cavity length is $\approx 150\mu$. The phase coherence length should be much larger than $1/\Omega_c$ to realize a system that is capable of performing non trivial operations. The best reported values for decoherence times in a quantum dot system are comparable to $1/\Omega_c$. [16] These dots were however open in the sense that large electron reservoirs were connected to them. Isolated quantum dots that are specifically designed to reduce the decoherence times will be of paramount importance, not only here but also in other applications of coherent phenomena.

Measurement of the final state of the quantum computer is crucial for an experiment. In this proposed model, qubits are inside the cavity, the state of a qubit can be measured by the following procedure: (a) Transferring the quantum state of the qubit to the cavity by bringing the qubit into resonance with the cavity and waiting for half the time period of the atom-cavity Rabi oscillation. If the electron in the qubit is in higher state, it will release a photon to the cavity. (b) This photon has to be detected from the cavity by a detector, which is a difficulty that all models of quantum computing suffer from.

Here we have shown a new model of quantum-computer using atoms or quantum dots inside a quantum cavity. A similar model can be easily designed for spin-states inside a cavity by replacing the Stark effect by a Zeeman effect. With the advance of technology, it may be possible to fabricate steady atoms inside the cavity or quantum dots inside a cavity with long enough decoherence time. The important point of this model is that the qubits are easily addressed (and we don't require a separate laser addressing each one). Note that operations are done only when the cavity/laser and the atomic levels are on-resonance, while undesired interactions are avoided by keeping the far off-resonance condition for the other atoms. The main operations are done by an external laser and controlling the voltage of the stark plates from outside.

* Present address: 111 Dana Research Center, Department of Physics, Northeastern University, Boston, MA, 02115. E-mail: pradhan@neu.edu.

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